Portfolio Optimisation and Efficient Frontier

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Group project. Collaborator names and student identifiers are omitted from this public copy.

Introduction

This report examines whether it is possible to construct a portfolio of four U.S. stocks, Chipotle Mexican Grill (CMG), O'Reilly Automotive (ORLY), Monster Beverage (MNST), and Tesla (TSLA), that achieves a benchmark annual return of 8% with the probability of underperformance below 20%. We perform a mean variance analysis using five years of weekly price data (01/01/2021 to 31/12/2025), assuming jointly normal returns.

Data & Methodology

Weekly closing prices were retrieved via Google Finance for the period 8 January 2021 to 31 December 2025. Weekly returns and covariances were computed and then averaged and annualised by multiplying by 52, which is valid under the assumption of independent, identically distributed weekly returns. No dividend adjustments were required as none of the four stocks paid dividends during the period. Table 1 and table 2 summarises the annualised expected returns and covariances between the stocks respectively.

Table 1: Average Returns Calculation				
Ticker	CMG	ORLY	MNST	TSLA
Average Weekly Returns	0.22%	0.46%	0.24%	0.50%
Annualised Weekly Returns	11.53%	23.96%	12.34%	26.19%

Table 2: Annualized Covariance Calculations				
	CMG	ORLY	MNST	TSLA
CMG	0.110478	0.014177	0.015476	0.053197
ORLY	0.014177	0.053335	0.011872	0.010792
MNST	0.015476	0.011872	0.052104	0.025980
TSLA	0.053197	0.010792	0.025980	0.339938

TSLA exhibits the highest expected return (26.19%) but also the greatest volatility, while ORLY offers a strong return (23.96%) with substantially lower risk. The covariance matrix highlights key diversification dynamics. TSLA exhibits the highest variance (0.3399) and strongest co-movement with CMG (0.0532), meaning portfolios tilted toward these two assets carry substantial risk. ORLY, by contrast, has the lowest covariances with all other stocks, particularly TSLA (0.0108), making it the most effective diversifier.

Mean variance Analysis & Efficient Frontier

We solved the minimum-variance optimisation problem for target expected returns ranging from 11.83% to 26.00% (in 0.5% increments), subject to the constraint that portfolio weights sum to one (allowing for the possibility of short selling). The frontier (as shown in figure 1) exhibits the characteristic convex "bullet" shape predicted by Markowitz portfolio theory. The lower portion of the frontier represents inefficient portfolios, while the upper branch constitutes the set of efficient portfolios offering the highest expected return for a given level of risk. The global minimum-variance portfolio (GMVP) has an expected return of 17.33% and a standard deviation of 17.17%. Target returns below 17.33% lie on the lower (inefficient) portion of the frontier and are dominated by the GMVP, which achieves the same or lower risk at a higher return. While this portfolio minimises risk, it does not necessarily satisfy the supervisor's downside risk requirement, which depends jointly on both expected return and volatility. Table 3 shows the minimum variance portfolio and table 4 shows the weights of each stock.

Table 3: Min variance Portfolio	
Return	17.33%
Variance	0.029488
Standard Deviation	17.17%
Target Return	17.33%
Underperformance Probability	29.35%
Z score	0.54

Table 4: Minimum Variance Portfolio stock weights	
Ticker	**Weight**
CMG	14.57%
ORLY	41.74%
MNST	41.82%
TSLA	1.88%

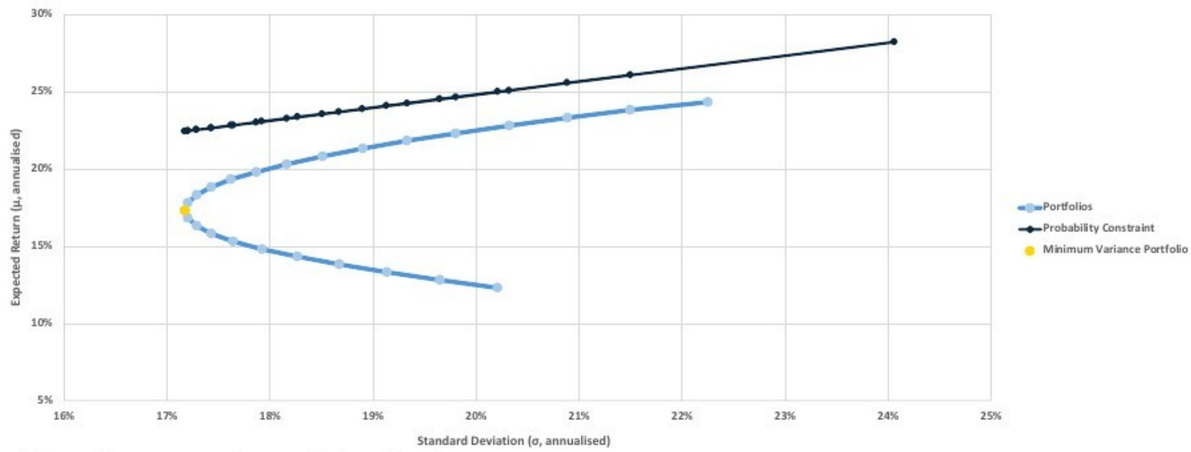

Figure 1: mean-variance efficient frontier

Feasibility Assessment

Under the normality assumption, the probability of the portfolio return R_p falling below the benchmark B = 8% is given by:

$$P\left(R_p < B\right) = \Phi \frac{\mathrm{B} - \mu_\mathrm{p}}{\sigma_p}$$

where Φ is the standard Normal CDF. The requirement $P\left(R_p < 0.08\right) < 0.20$ is equivalent to requiring the z-score to satisfy $z < -0.8416$, which is the 80th percentile of the standard normal distribution. This defines a linear boundary in mean standard deviation space, only portfolios lying above the line $\mu = 0.08 + 0.8416\sigma$ satisfy the constraint. As shown in Figure 1 and the Appendix, the minimum underperformance probability across the entire efficient frontier is 23.07%, achieved at $\mu = 23.83\%$ and $\sigma = 21.50\%$ which falls short of the 8% threshold. Since no point on the frontier reaches or crosses this boundary, the constraint is infeasible for this asset universe.

Table 5: Portfolio with Min Underperformance Probability	
Ticker	**Weight**
CMG	0.00%
ORLY	80.59%
MNST	7.65%
TSLA	11.76%

The minimum underperformance probability of 23.07% is achieved at the efficient portfolio with an expected return of approximately 23.83% and a standard deviation of 21.50% with the corresponding weights shown in table 5. The heavy allocation to ORLY reflects its attractive risk return profile and low covariance with the other assets.

Consequently, no portfolio constructed can satisfy both the return requirement and the downside risk constraint. Even portfolios with relatively high expected returns exhibit levels of volatility that result in an unacceptably high probability of underperformance under the assumed normal distribution. The GMVP, despite having the lowest attainable risk, also violates the constraint, as its expected return is insufficient to offset its volatility in probabilistic terms.

Conclusion

This analysis finds that it is not possible to construct a portfolio that meets the supervisor's requirement of achieving an expected annual return of at least 8% while keeping the probability of underperformance below 20%. The underperformance probability constraint is binding and lies above the entire mean-variance efficient frontier.

This result highlights the importance of considering downside risk alongside expected return and volatility. While the asset universe offers portfolios with attractive expected returns, their associated risk levels imply substantial downside probabilities relative to the benchmark. Meeting the supervisor's requirement would therefore necessitate either a different asset universe or a relaxation of the downside risk constraint.

Appendix

Efficient Frontier full results

Table 6: Efficient Frontier Table			
Target Return	**Min Var**	**Standard Deviation**	**Underperforming Probability**
11.8%	0.057896	24.06%	43.68%
12.3%	0.040838	20.21%	41.52%
12.8%	0.038596	19.65%	40.29%
13.3%	0.036604	19.13%	39.03%
13.8%	0.034860	18.67%	37.74%
14.3%	0.033365	18.27%	36.45%
14.8%	0.032120	17.92%	35.16%
15.3%	0.031123	17.64%	33.89%
15.8%	0.030376	17.43%	32.66%
16.3%	0.029876	17.28%	31.49%
16.8%	0.029585	17.20%	30.38%
17.3%	**0.029488**	**17.17%**	**29.35%**
17.8%	0.029586	17.20%	28.38%
18.3%	0.029879	17.29%	27.50%
18.8%	0.030365	17.43%	26.71%
19.3%	0.031046	17.62%	26.01%
19.8%	0.031921	17.87%	25.39%
20.3%	0.032991	18.16%	24.86%
20.8%	0.034255	18.51%	24.41%
21.3%	0.035713	18.90%	24.03%
21.8%	0.037366	19.33%	23.72%
22.3%	0.039213	19.80%	23.46%
22.8%	0.041291	20.32%	23.28%
23.3%	**0.043622**	**20.89%**	**23.15%**
23.8%	0.046207	21.50%	23.07%
24.3%	0.049508	22.25%	23.15%
24.8%	0.076880	27.73%	27.19%
25.3%	0.141400	37.60%	32.24%
25.8%	0.243069	49.30%	35.88%
26.0%	0.339938	58.30%	37.75%