Unsymmetrical Bending of a Cantilever Beam

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Group project. Collaborator names and student identifiers are omitted from this public copy.

Introduction

Outline

Firstly, bending is the distortion or curvature that happens in a structural component as a result of applied external forces or moments. There are 2 main types of bending, symmetric or unsymmetrical, with each having particular characteristics and effects.

Symmetric bending, is when the plane of loading or bending is co-incident with or parallel to a plane containing the fundamental centroidal axis of inertia of the beam's cross-section. (Unsymmetrical Bending(2023)) The applied stresses or bending moments are distributed symmetrically, causing bends without twisting. This ensures that the internal resistive moments are aligned with the applied external moments, making analysis easier.

Unsymmetrical bending occurs when the plane of bending does not coincide with or parallel to the plane containing the centroidal axis of the cross section. As a result, stress is distributed in an inconsistent pattern. () Because of the asymmetric sectional form of the beam and the fact that the loads acting on the beam are not in the perpendicular axis in the plane of bending, unsymmetric bending is more important and is often utilized in real life. The force or bending moment must be resolved into components that act along the primary axes in order to evaluate stresses and deflection of the beam. Unsymmetrical bending is needed in engineering as we need to understand complicated issues in structural analysis and design. The non-linear stress distribution and the combined effect of bending and torsion contribute to the complexity, necessitating a more nuanced and rigorous analytical approach. This understanding is critical for engineers and architects when designing and testing the stability, robustness, and resilience of various structural parts to ensure their performance under a variety of and unforeseen loading conditions.(Jeffery S. et al (2020)) Unsymmetrical bending can be used in many fields from the aviation industry to the construction industry. Engineers need to calculate the maximum deflection and therefore loads a structure can be subjected to before it would fail.

So, the aims of this experiment was to:

1. To verify that a cantilever beam specimen remains in the linearly elastic range for a range loads.
2. To measure the vertical and horizontal deflections of the cantilever beam specimen loaded in the direction of a principal axis.
3. To measure the vertical and horizontal deflections of the cantilever beam specimen loaded at angles other than in the direction of a principal axis (off-axis loading).
4. To compare the experimentally obtained beam deflections to the theoretical ones.

Background and theory

During our experiment we used 3 different beam configurations, which were 0°, 30°, 60° in that order respectively. In the initial configuration of the beam at 0°, the deflection of the beam in the y-direction could be found using the equation below:

$$\delta = \frac{-FL^3}{3EI} \tag{1.1}$$

Where, δ is the force applied to the beam, L is the length of the beam, E is the Young's modulus of the beam, and I is the second moment of area of the beam's cross section. The young modulus of the beam was given to us at a constant of 170 GPa.

For the deflections to be found in unsymmetrical bending, due to the beam is turned clockwise across the 3 different beam configurations, the force must be resolved into components u and v as they act along the principal axes. However, the moment of inertia should be found in the x and y component for it to be resolved.

$$I_{xx} = \frac{bh^3}{12} \qquad I_{yy} = \frac{b^3h}{12} \tag{1.2) \& (1.3}$$

Where, the notation b and h are base and height respectively of the beam's cross section. To find the new second moment of area in the resolved components of u and v, we can use the following equations for finding I about its respective axes:

$$I_{UU} = I_{xx}(\cos\theta)^2 - 2(cos\theta)(\sin\theta) + I_{YY}(\sin\theta)^2 \tag{1.4}$$
$$I_{VV} = I_{yy}(\cos\theta)^2 - 2(cos\theta)(\sin\theta) + I_{xx}(\sin\theta)^2 \tag{1.5}$$

When derived, the new second moment of area in the resolved components, it can be put back into equation (1.1), to determine the horizontal and vertical deflections.

$$\delta_x = -\frac{Fsin\theta L^3}{3EI_{yy}} \qquad \delta_y = -\frac{Fcos\theta L^3}{3EI_{xx}} \tag{1.6) \& (1.7}$$

Lastly, the following equations can be used to derive the real deflection of the beam as trigonometric components of the deflections determined above.

$$\Delta v_x = \delta_y cos\theta - \delta_x sin\theta \qquad \Delta v_y = \delta_y sin\theta - \delta_x cos\theta \tag{1.8) \& (1.9}$$

Results

Sample calculation (horizontal deflection at 30 degrees)

$$\delta_x = \frac{-FL^3}{3EI}$$
$$I_{UU} = I_{xx}(\cos\theta)^2 - 2(cos\theta)(\sin\theta) + I_{YY}(\sin\theta)^2$$

$$I_{UU} = 2813.17$$

$$\delta_x = \frac{-5(515)^3}{3(170000)(2813.17)}$$

$$\delta_x = -0.027\ mm$$

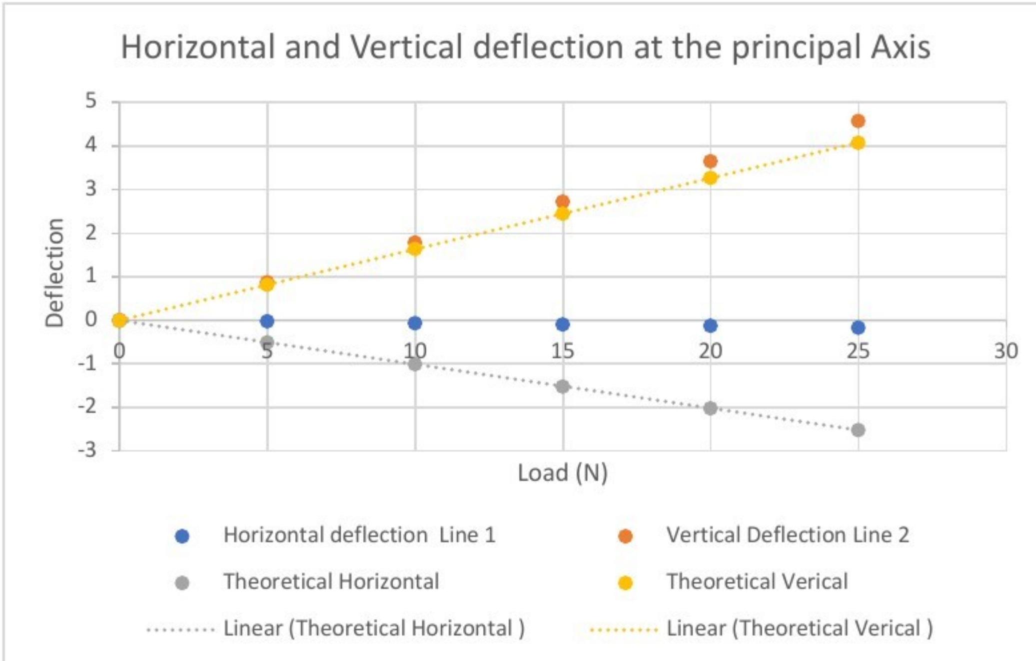

Figure 1 – Graph of the theoretical and actual horizontal and vertical deflection at the principle axis

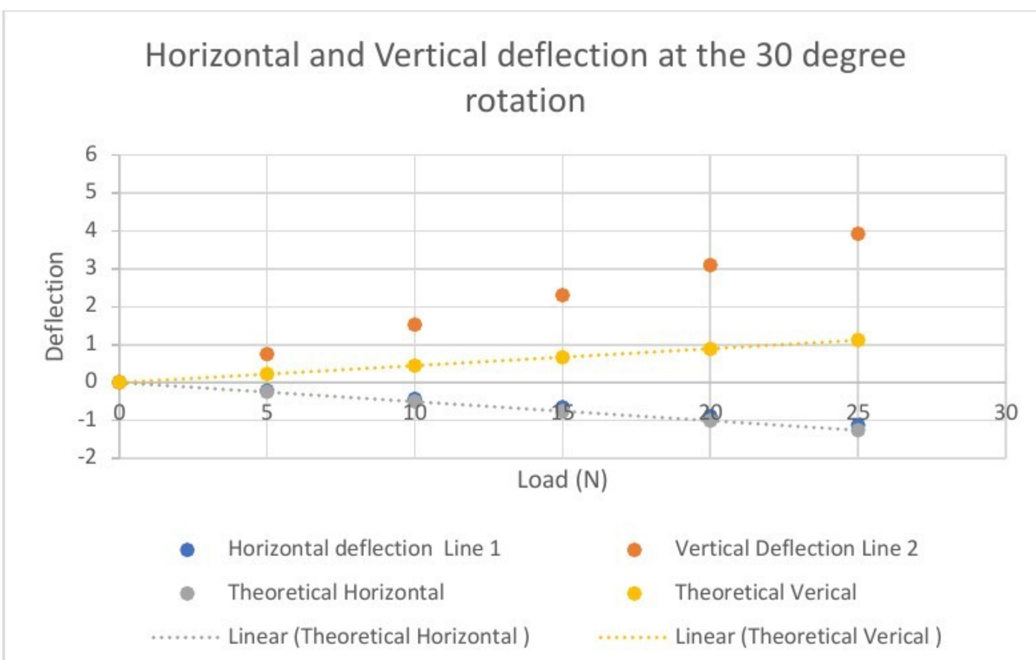

Figure 2 – Graph of the theoretical and actual horizontal and vertical deflection at 30 degrees rotation clockwise

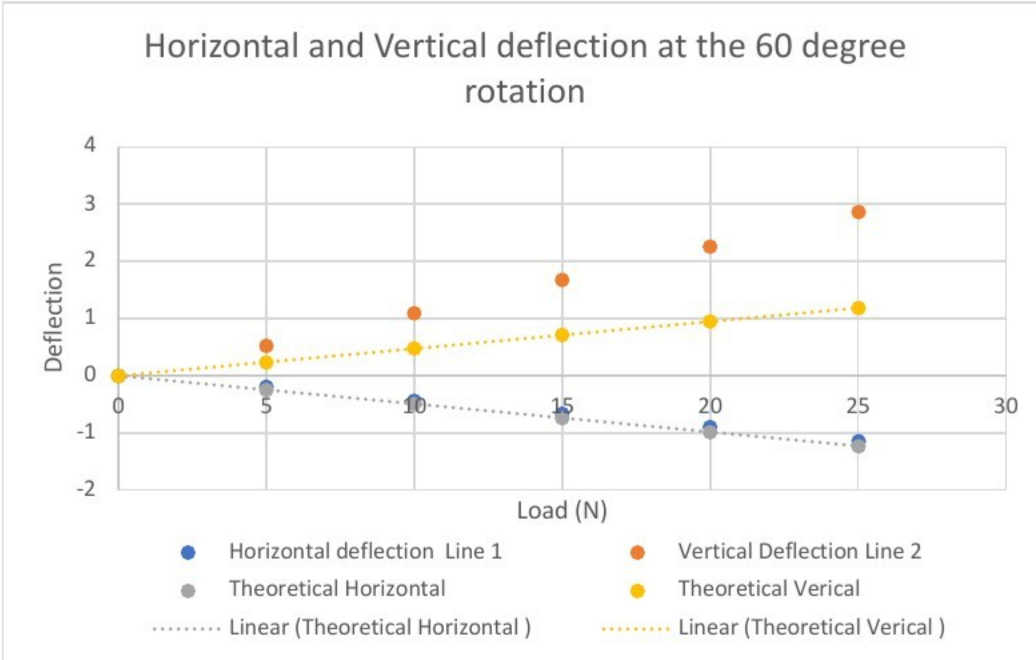

Figure 3 – Graph of the theoretical and actual horizontal and vertical deflection at 60 degrees rotation clockwise

Discussion

In this experiment, our aims were to measure the deflection of a beam in both the vertical and horizontal planes, with the beam loaded in the direction of the principal axis and off the principal axis (off-axis loading) whilst ensuring that the beam remained in the linearly elastic range throughout the experiment. Another aim was to compare the experimentally obtained beam deflection to the theoretical ones.
The experiment was successful, but our team did encounter some difficulties whilst doing it. The deflection gauges were accurate to two decimal planes (millimetre smallest measurable unit) making them extremely sensitive to any change in the position of the beam so when the loads were removed from the beam, the gauge would sometimes not be zeroed, and we would have to reset it. Overall, this was not a huge issue and would have negligibly affected out results.
Apon analysis, we discovered trends in our data. The trends were:

1. The higher the load the greater the deflection
2. Maximum deflection in the Y-axis occurred when the Y-axis was parallel to the vertical axis.
3. Maximum deflection in the X-axis occurred when the X-axis was most parallel to the vertical axis.

All these trends are fully supported by the theory and were expected. The only discrepancy in the results we obtained and the theory was that when the Y-axis was fully parallel to the vertical axis, there was still a deflection in the X-axis. We thought this could have been caused because of using weights that were not symmetrical. This means their centres of masses were not perfectly centred, and this would cause a small but unwanted X component in the weight force of the

applied load. Our team also decided to run the test 3 times and to average our results, as we realised that using these weights would introduce a random error to all of results.
If we were to conduct this experiment again in the future, we would use centred masses and gathered data for more angles of orientation to easier see trends when rotating the beam off its principal axis.

Conclusion

From the experiment conducted, we successfully gathered deflection measurements for loads applied to the end of a beam. From our results we see that the beam stays in linearly elastic range. The deflection measurements gathered were supported by the theory, both for off-axis loading and principle axis loading. Random errors were caused by the uncentred weights which showed in the results. There were further small noticeable errors in the deflection when the beam was touched, when adding weights it was necessary for us to touch parts of the experiment which may have arises to small errors in the experimental values.

References

1. Unknown, (2023) Unsymmetrical Bending https://ecajmer.ac.in/facultylogin/announcements/upload/Unsymmetrical%20Bending.pdf

2. Thomas, Jeffery S. et al (2020) *Mechanics of Materials*. 5th edition: Wiley-Blackwell

Appendix

I_y =	6371.144	6371.14	Deflection			
I_x =	1641.667		y	0 degree	30 degree	60 degree
I_u30 =	2823.17	31531.37	0	0	0	0
I_u60 =	5187.909	33881.37	5	0.81571141	0.223467745	0.237021708
I_v30 =	5189.641	33881.37	10	1.63142281	0.44693549	0.474043416
I_v60 =	2824.902	31531.37	15	2.44713422	0.670403235	0.711065124
			20	3.26284563	0.89387098	0.948086832
E =	170000		25	4.07855703	1.117338724	1.18510854
L =	515					
			x			
			0	0	0	0
			5	-0.4743342	-0.237167124	-0.2458411
b =	19.7		10	-0.9486685	-0.474334248	-0.491682201
h =	10		15	-1.4230027	-0.711501371	-0.737523301
			20	-1.897337	-0.948668495	-0.983364401
			25	-2.3716712	-1.185835619	-1.229205502

	0			
X Axis	Horizontal deflection Line 1	Vertical Deflection Line 2	Theoretical Horizontal	Theoretical Verical
0	0	0.013	0	0
5	-0.027	0.88	-0.5036051	0.8157114
10	-0.063	1.787	-1.0072103	1.6314228
15	-0.1	2.73	-1.5108154	2.4471342
20	-0.13	3.65	-2.0144205	3.2628456
25	-0.167	4.57	-2.5180256	4.078557

X Axis	30			
X Axis	Horizontal deflection Line 1	Vertical Deflection Line 2	Theoretical Horizontal	Theoretical Verical
0	0	0	0	0
5	-0.2067	0.753	-0.251802563	0.223467745
10	-0.4267	1.52	-0.503605125	0.44693549
15	-0.6467	2.303	-0.755407688	0.670403235
20	-0.8867	3.093	-1.00721025	0.89387098
25	-1.1267	3.9267	-1.259012813	1.117338724

	60			
x axis	Horizontal deflection Line 1	Vertical Deflection Line 2	Theoretical Horizontal	Theoretical Verical
0	0	0	0	0
5	-0.193	0.523	-0.2458411	0.237021708
10	-0.4367	1.093	-0.491682201	0.474043416
15	-0.663	1.67	-0.737523301	0.711065124
20	-0.903	2.2567	-0.983364401	0.948086832
25	-1.14	2.863	-1.229205502	1.18510854